Tomographic velocity model estimation with data-derived first and second spatial traveltime derivatives

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Overview

- Introduction
- Velocity determination with CRS attributes
- A synthetic data example
- A real data example
- Extension to 3D
- Advantages/Limitations
- Conclusions

Introduction

- Problem: Determination of velocity model for depth imaging

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- Tomographic approach based on CRS stack results

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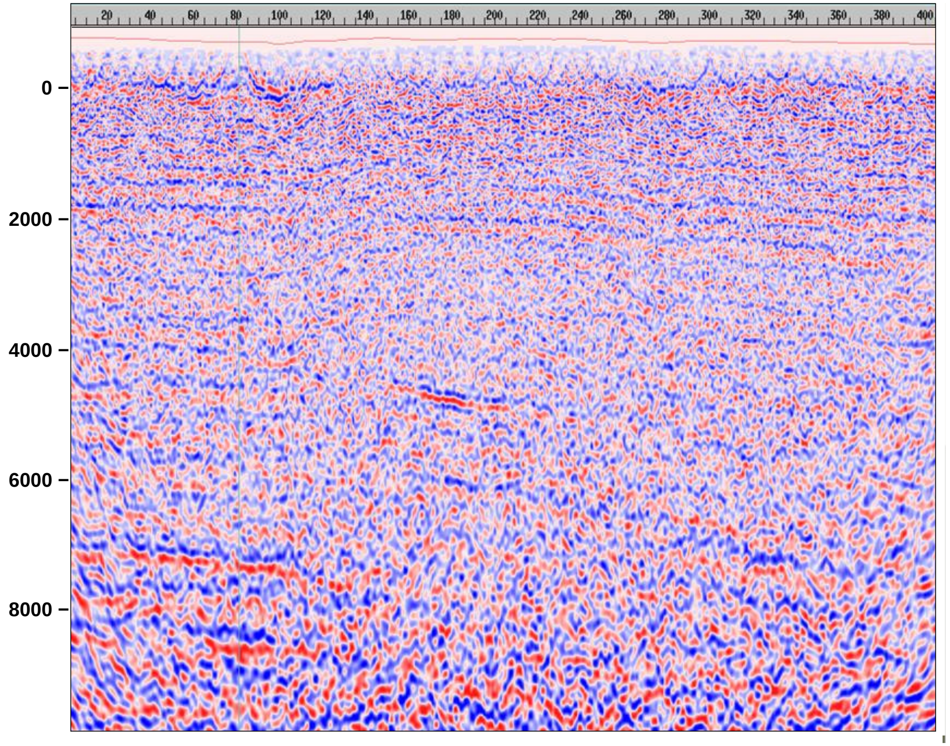
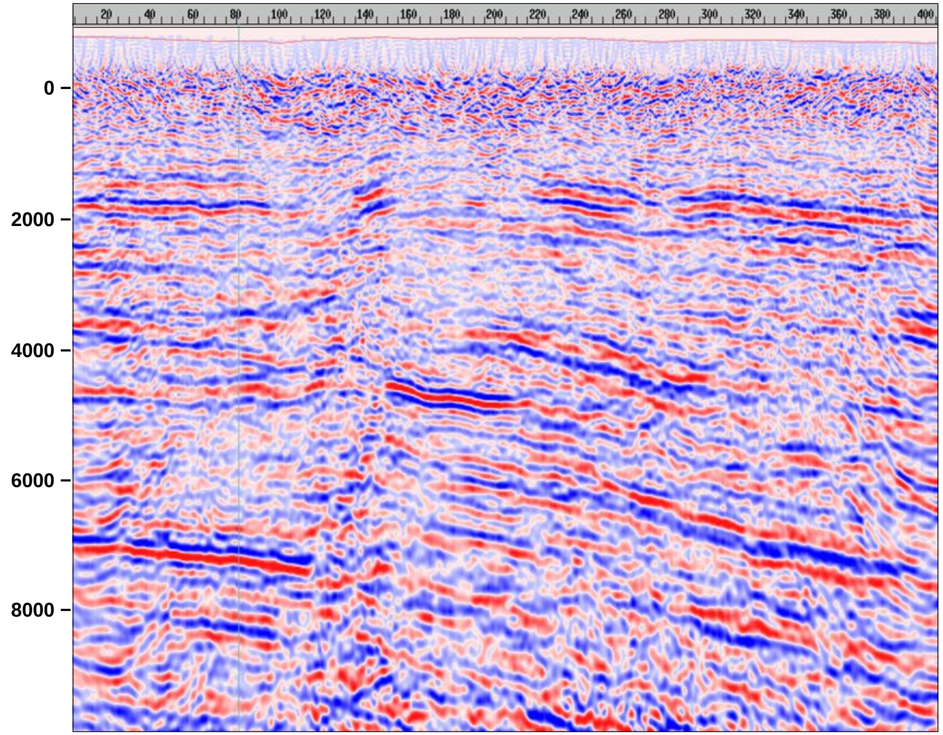
- Tomographic approach based on CRS stack results
- Smooth model description
- Advantages:
  - picking in simulated ZO section of high S/N ratio
  - pick locations independent of each other
    $\Longrightarrow$ very few picks required

CRS stack – 3D data example



PreSDM

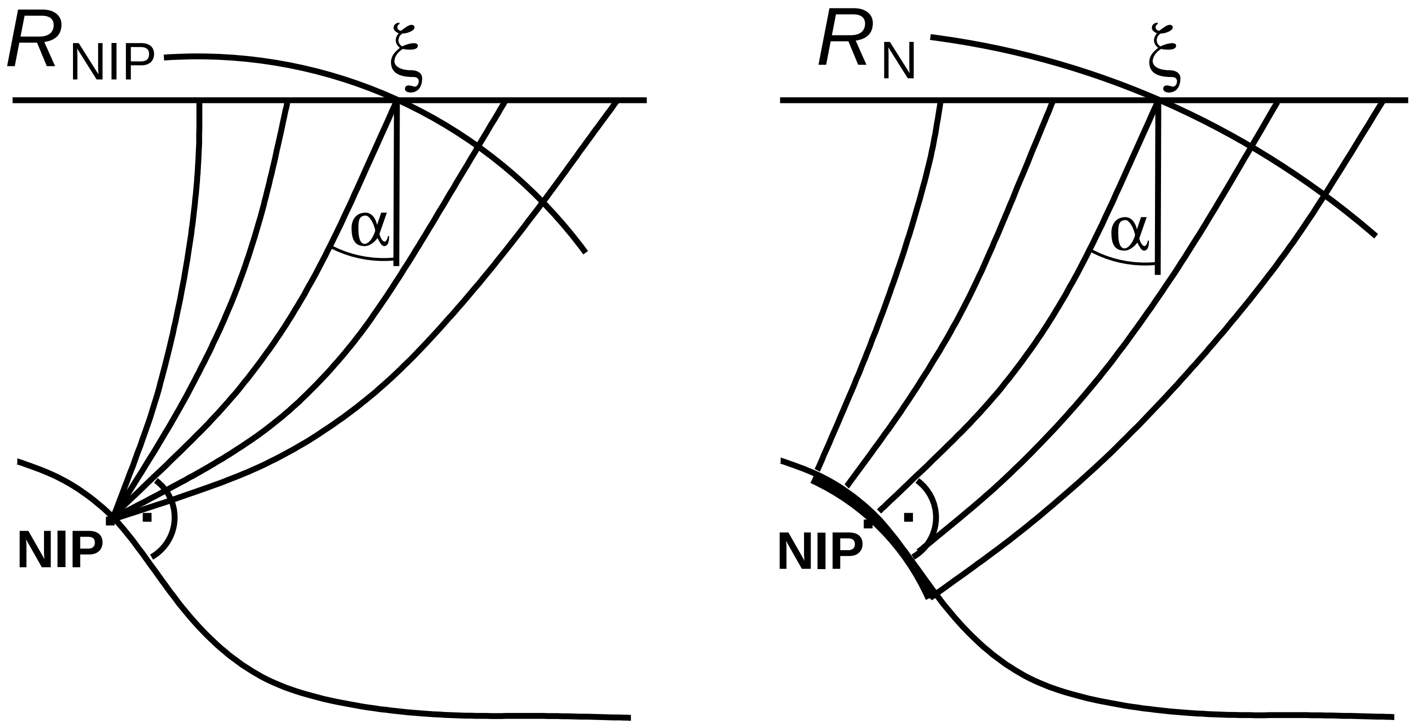


PostSDM of CRS stack

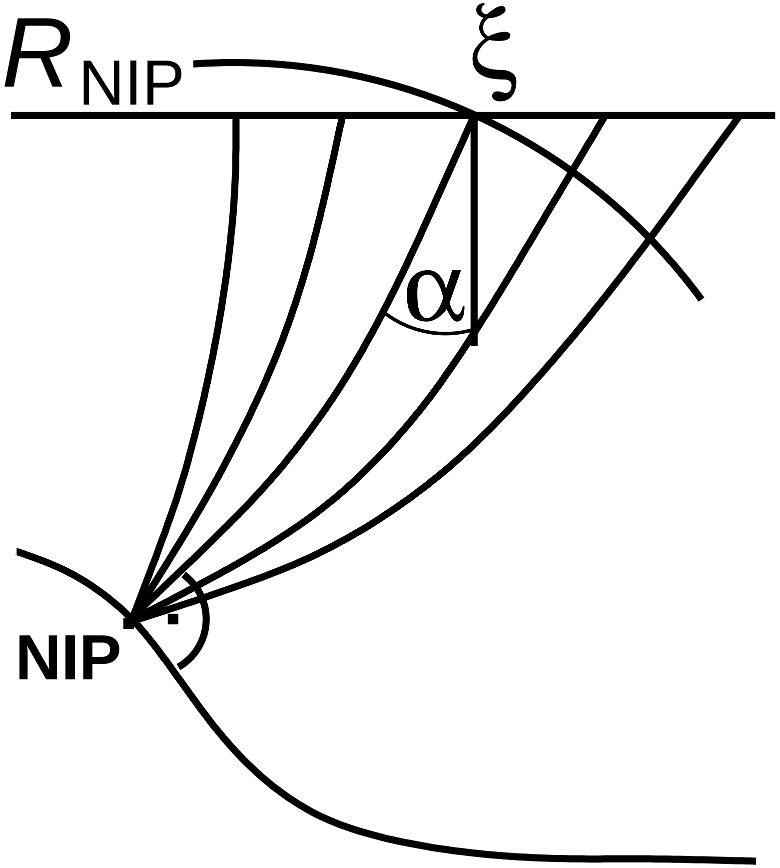
Data courtesy of ENI E&P Division

CRS stack and attributes

$$t^2(\xi_m, h) = \left(t_0 + \frac{2\sin\alpha}{v_0}(\xi_m - \xi)\right)^2 + \frac{2\,t_0\cos^2\alpha}{v_0}\left(\frac{(\xi_m-\xi)^2}{R_\mathrm{N}} + \frac{h^2}{R_\mathrm{NIP}}\right)$$



CRS attributes and velocities



- In the vicinity of a ZO ray: CRP-response can be approximately described by t_0, ξ, R_{NIP}, α

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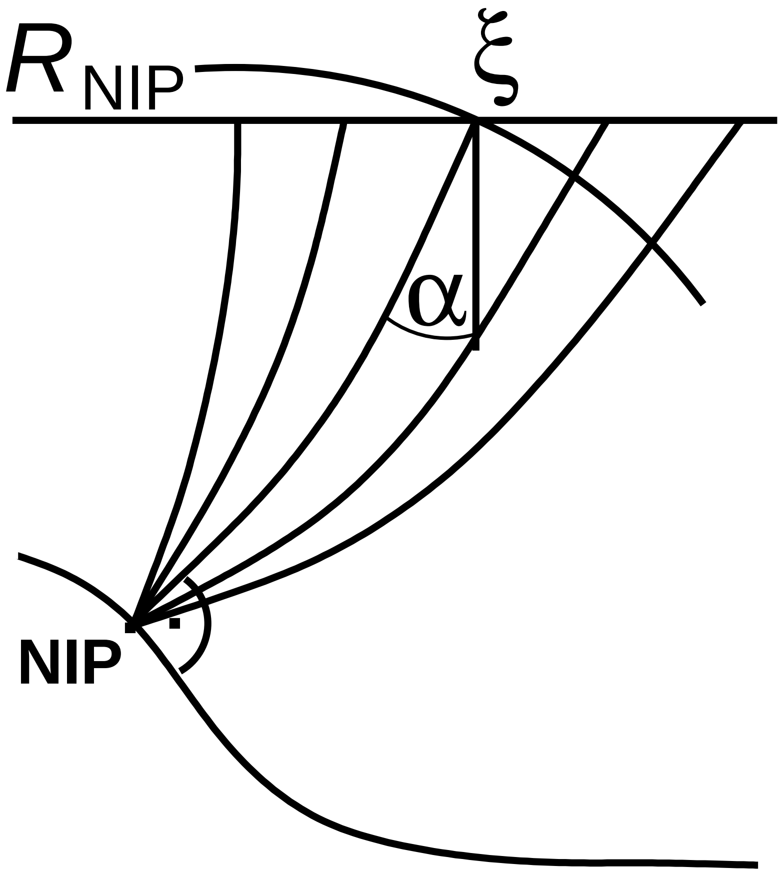
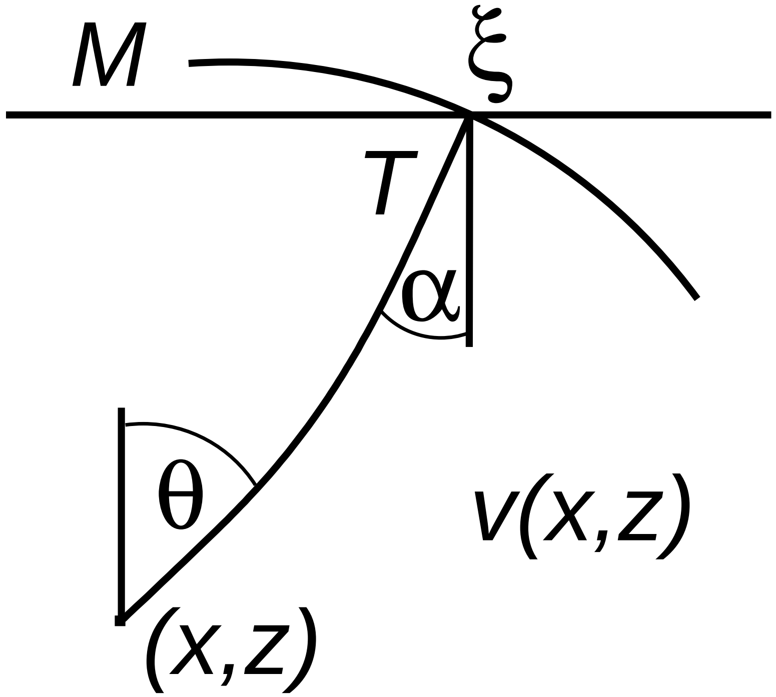


- In the vicinity of a ZO ray: CRP-response can be approximately described by $t_0,\ \xi,\ R_{\mathrm{NIP}},\ \alpha$
- Velocity model is consistent if $R_{\mathrm{NIP}} = 0$ at $t = 0$ for all considered data points

Tomography with CRS attributes

Data and model components



- Data: $(T, M, \alpha, \xi)_i$

$$M = 1/v_0 R_{\mathrm{NIP}}$$

$$T = t_0/2$$

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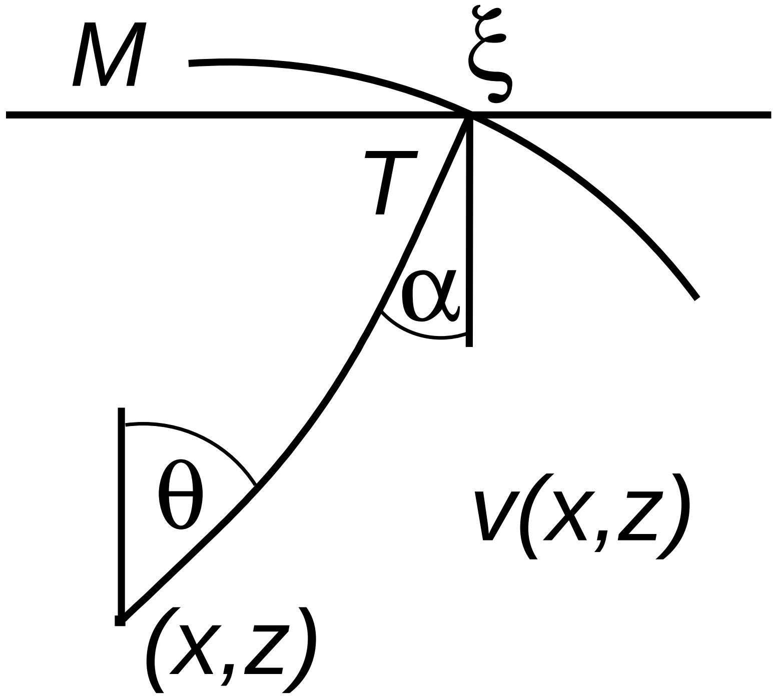

- Data: $(T, M, \alpha, \xi)_i$
- Model: $(x, z, \theta)_i, v_{jk}$

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v_{jk}: B-spline coefficients

Forward modeling

- Kinematic ray-tracing

$\Rightarrow T, \alpha, \xi$

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- Dynamic ray-tracing

  $\Rightarrow$ Ray propagator matrix $\mathbf{\Pi} = \begin{pmatrix} Q_1 & Q_2 \\ P_1 & P_2 \end{pmatrix}$

  $\Rightarrow M = P_2/Q_2$

Inversion procedure

- nonlinear least-squares problem

  $\Rightarrow$ iterative solution, linearize locally

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- model update $\Delta\mathbf{m}$: least-squares solution of

$$\mathbf{F}\Delta\mathbf{m} = \Delta\mathbf{d}$$

with $\Delta\mathbf{d}$: data misfit

$\mathbf{F}$: Fréchet derivatives

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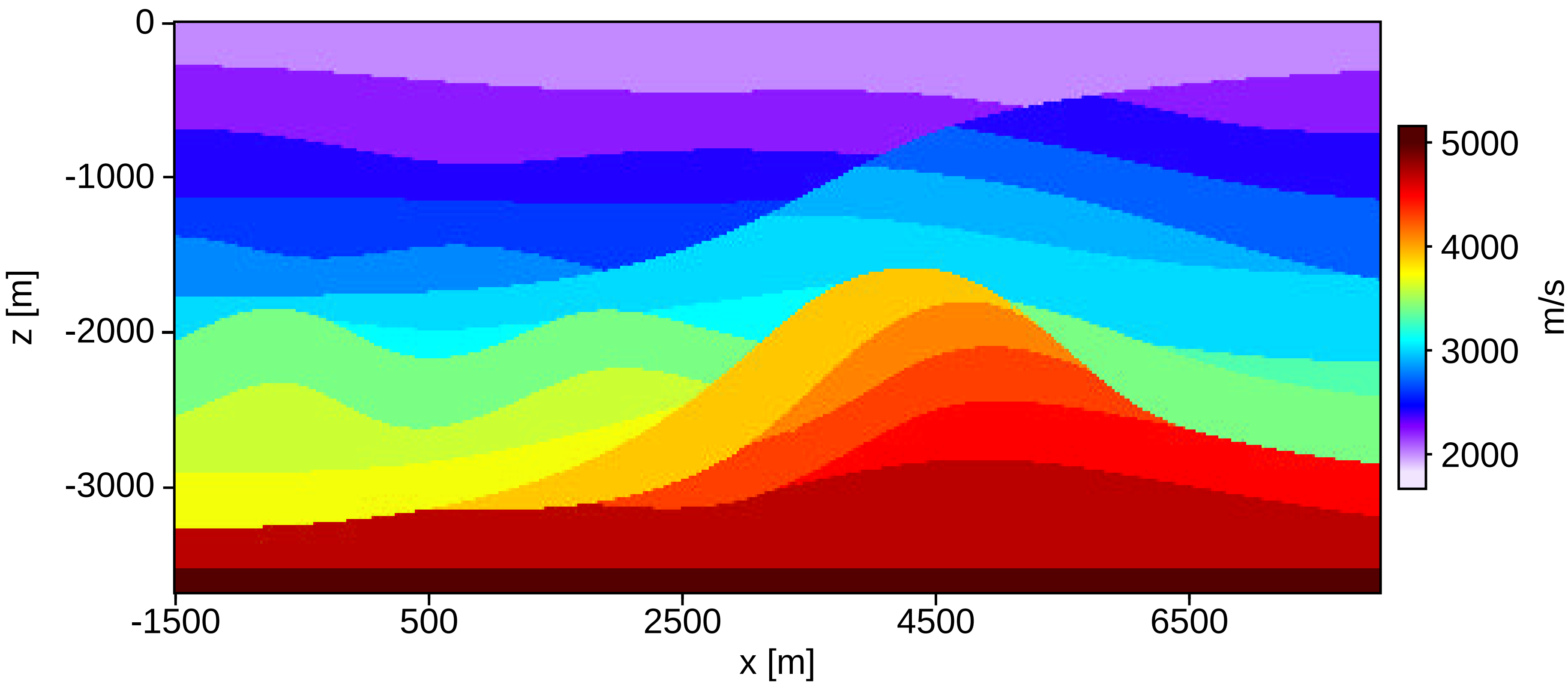
- regularization $\Rightarrow \hat{\mathbf{F}}\Delta\mathbf{m} = \Delta\hat{\mathbf{d}}$

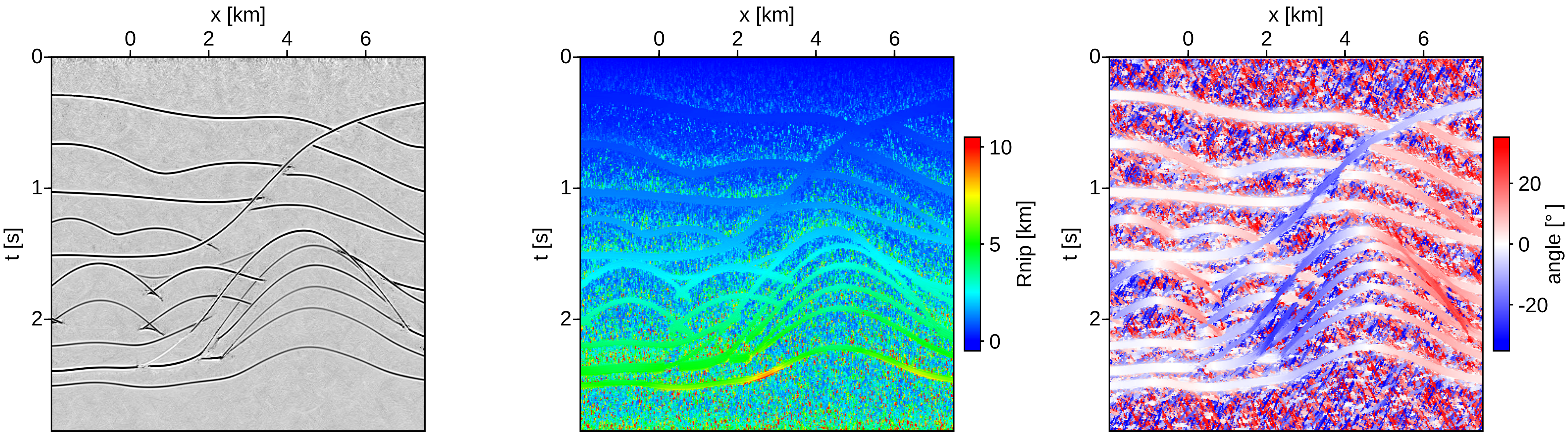
  (minimization of second derivatives of velocity)

A synthetic data example

Original velocity model



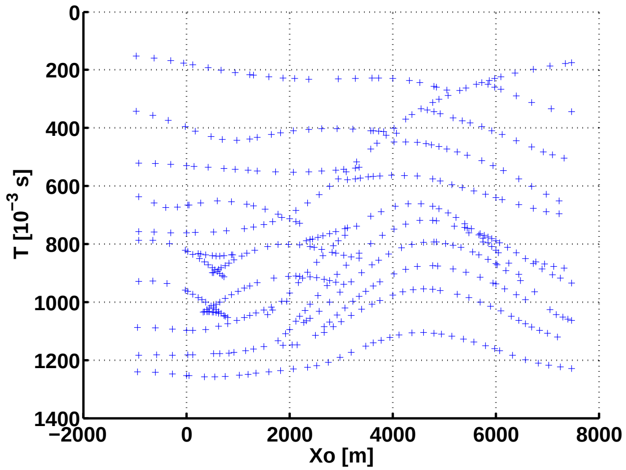
Synthetic data example



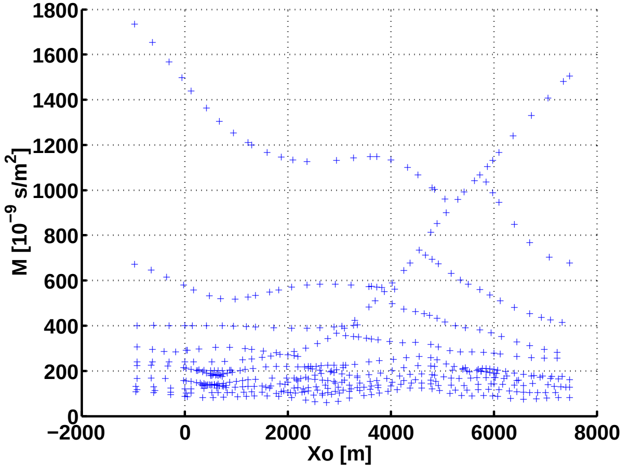
CRS stack $\qquad$ R_{NIP} section $\qquad$ α section

Synthetic data example

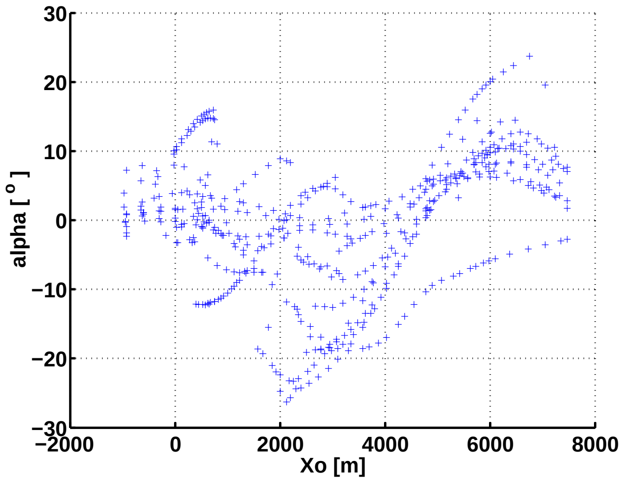
Picked input data for the inversion



T



M



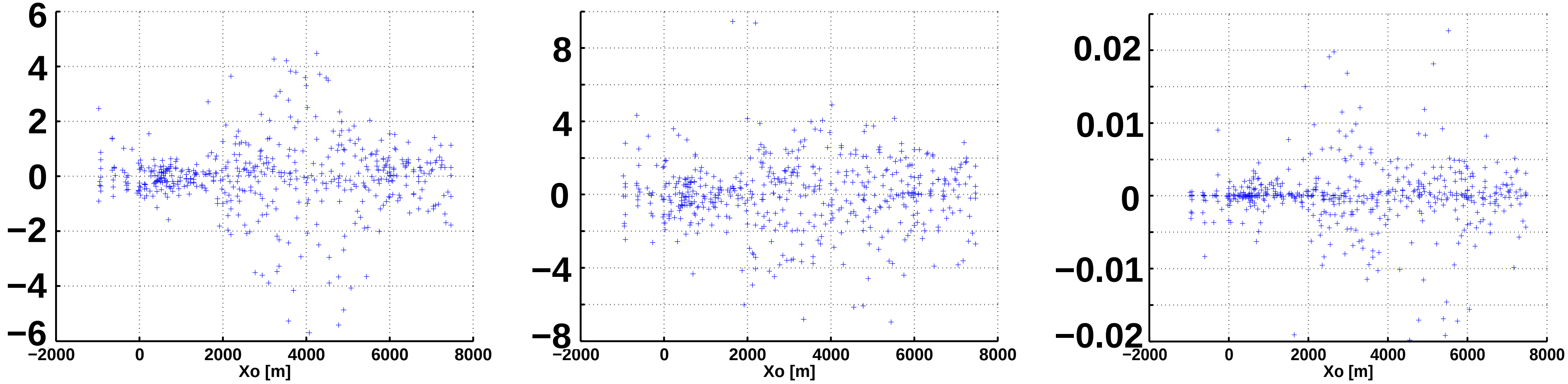
α

Model parametrization:

B-spline knot spacing Δx = 500 m, Δz = 300 m

Synthetic data example

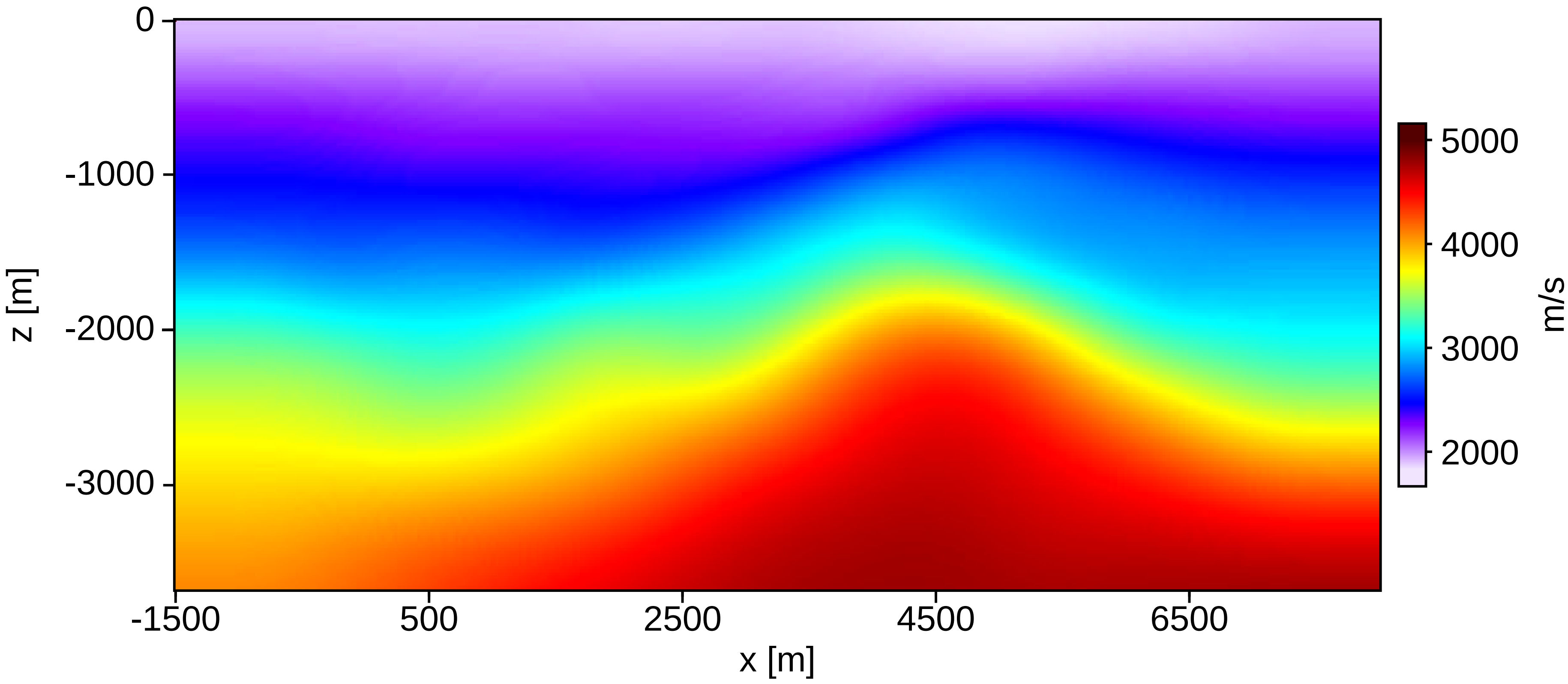
Residual data error after 12 iterations



$\Delta T\ [10^{-3}s]$ $\Delta M\ [10^{-9}\ s/m^2]$ $\Delta\alpha\ [^\circ]$

Synthetic data example

Inversion result



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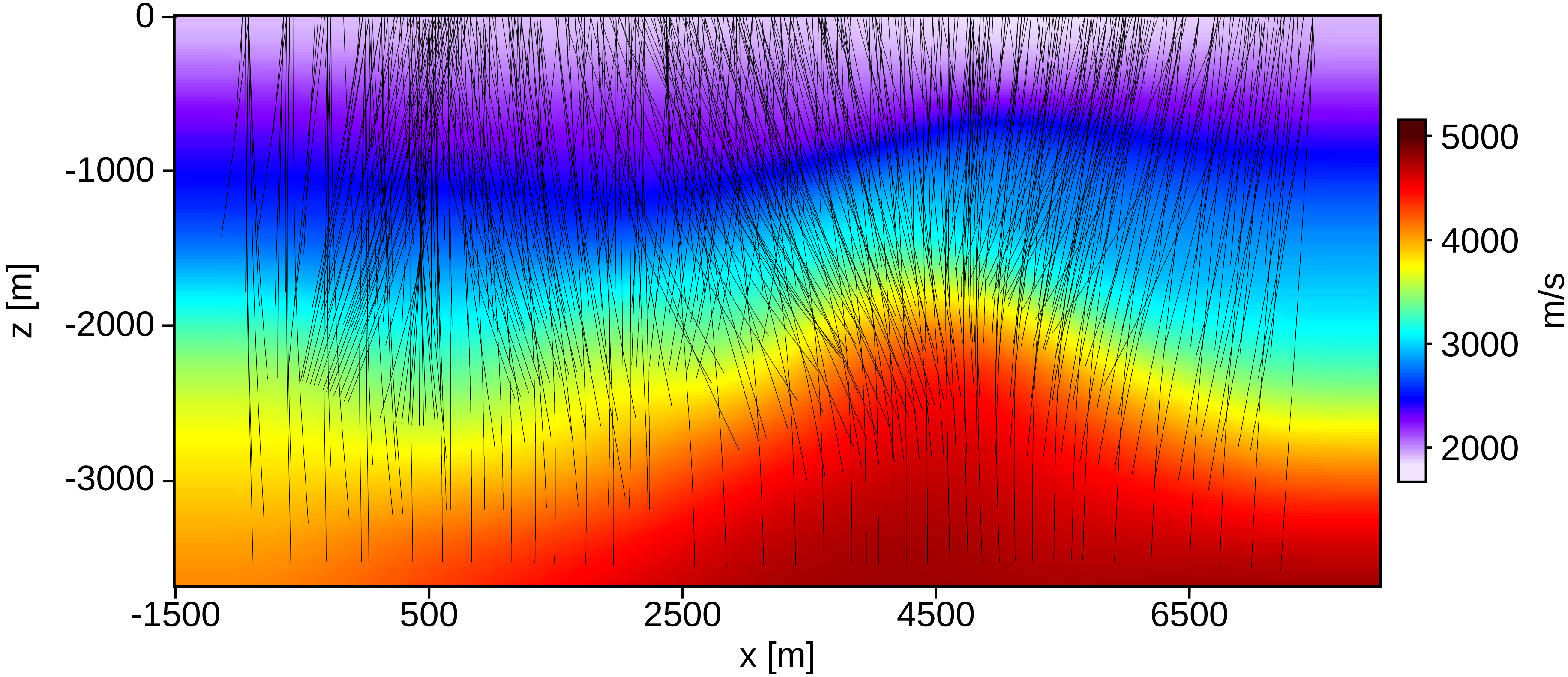
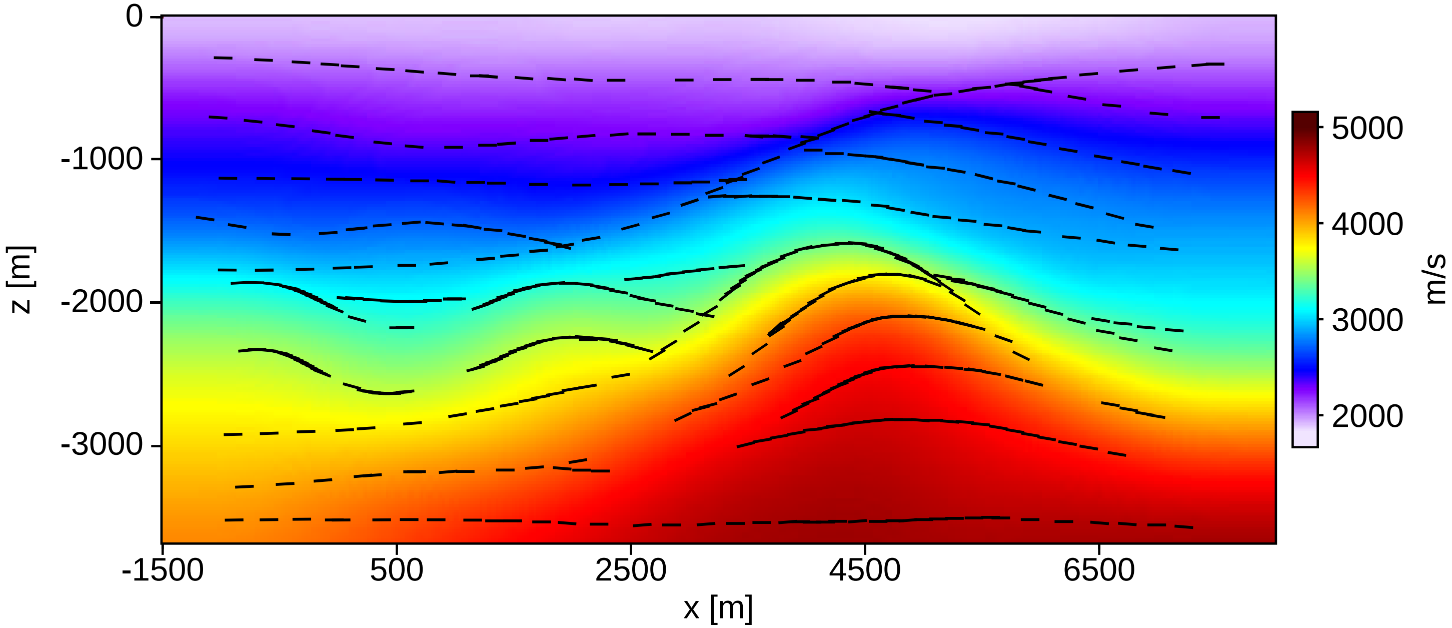
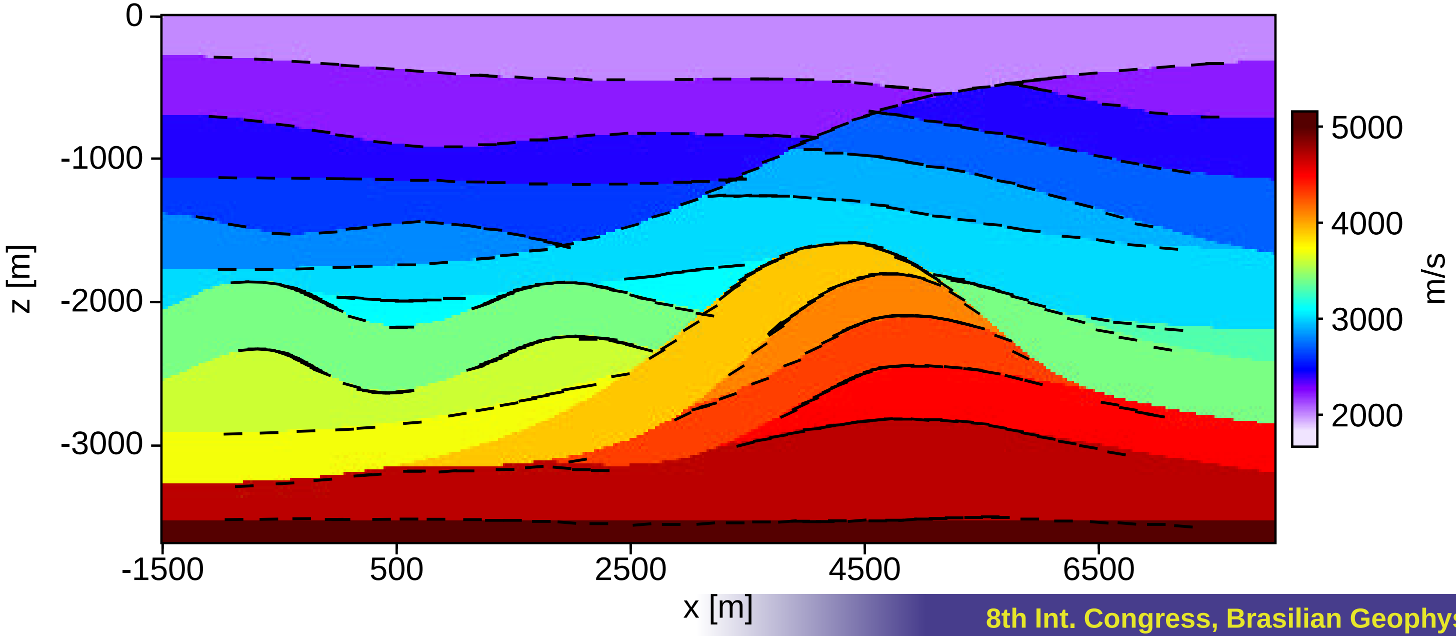


Reconstructed vs. original model

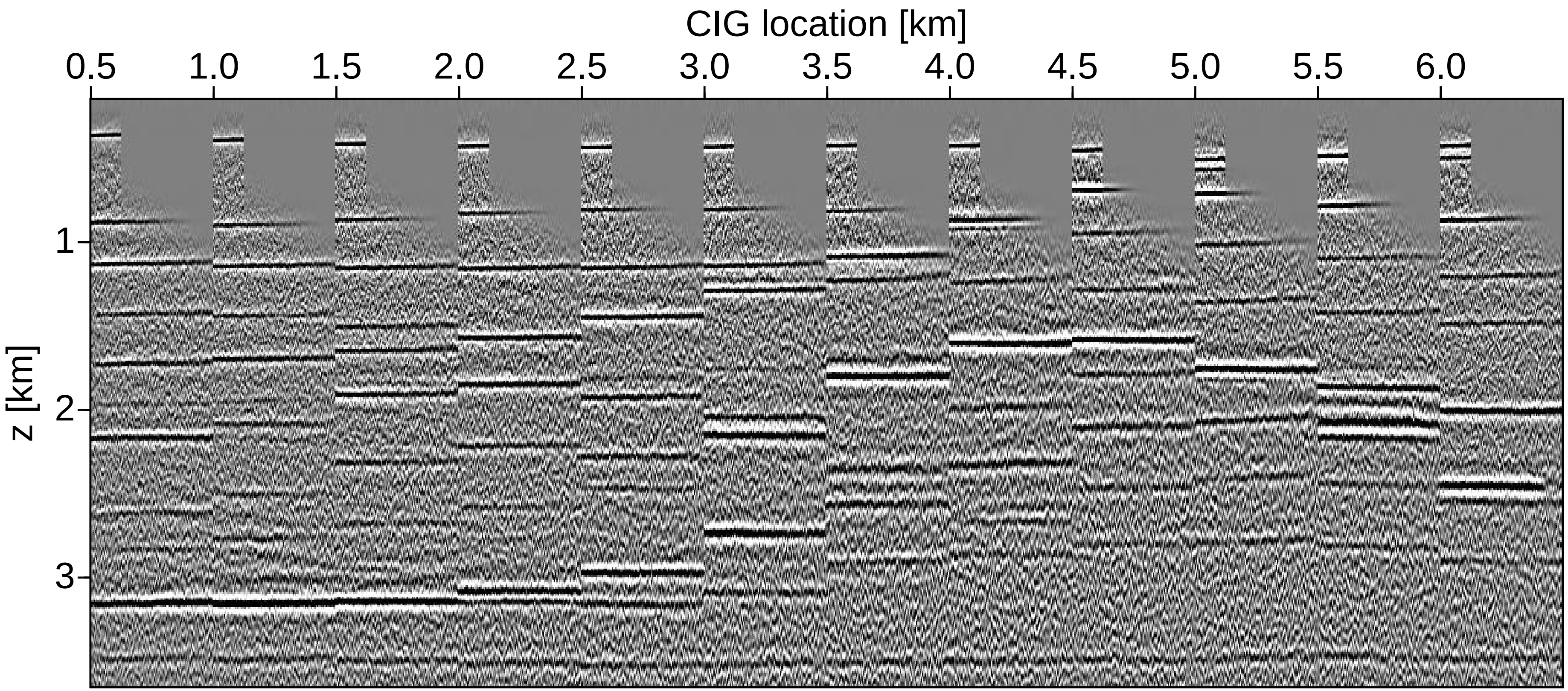
Reconstructed velocity and reflector elements



Original velocity and reconstructed reflector elements



Prestack migration results



common-image gathers (maximum offset=2000m)

Including additional constraints

. . . important in the case of data gaps!

- $v(x, z)$ values at arbitrary locations (x, z)

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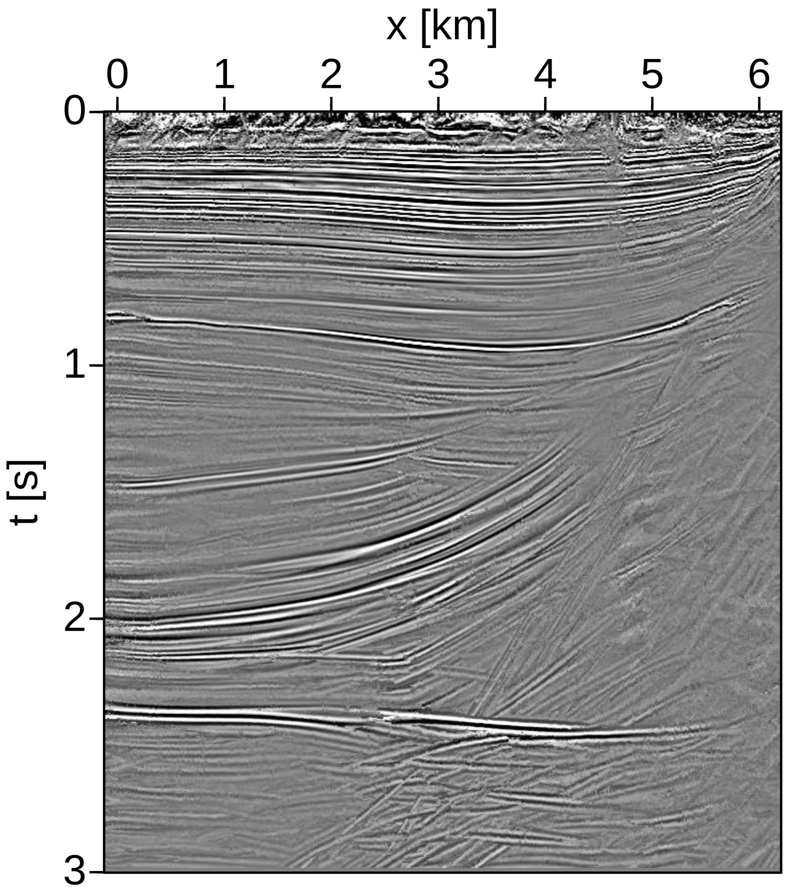
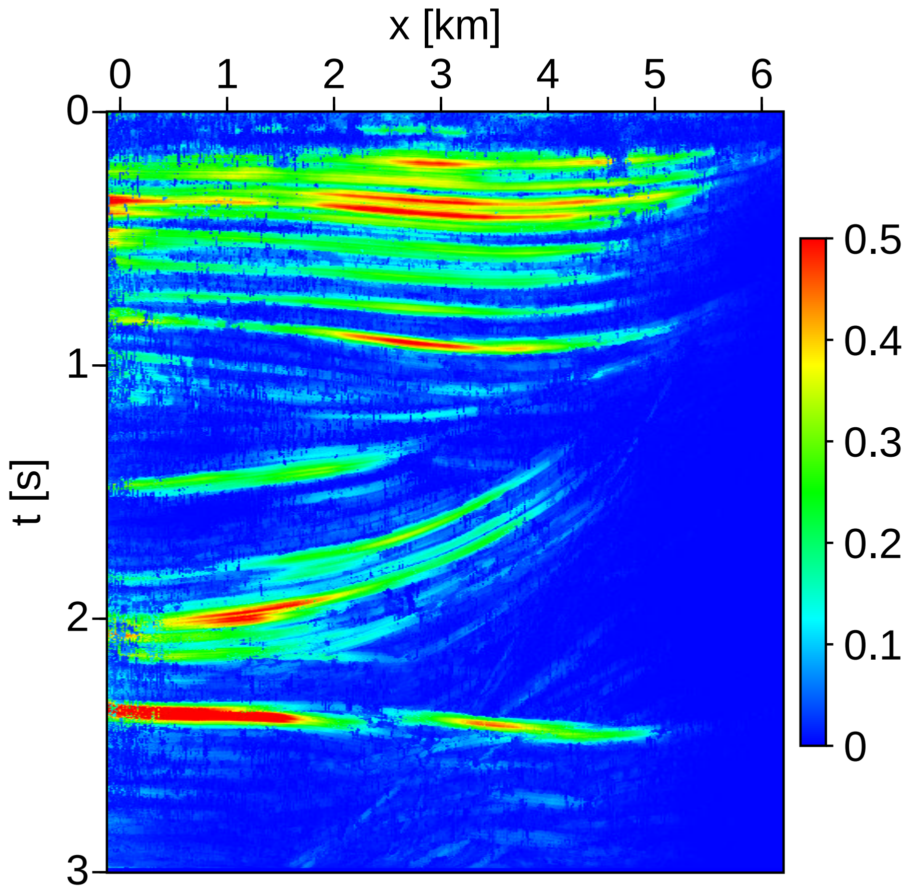
- $v(x,z)$ values at arbitrary locations (x,z)
- spatially dependent regularization (smoothness of velocity model)
- force velocity structure to follow local reflector structure

Real data example

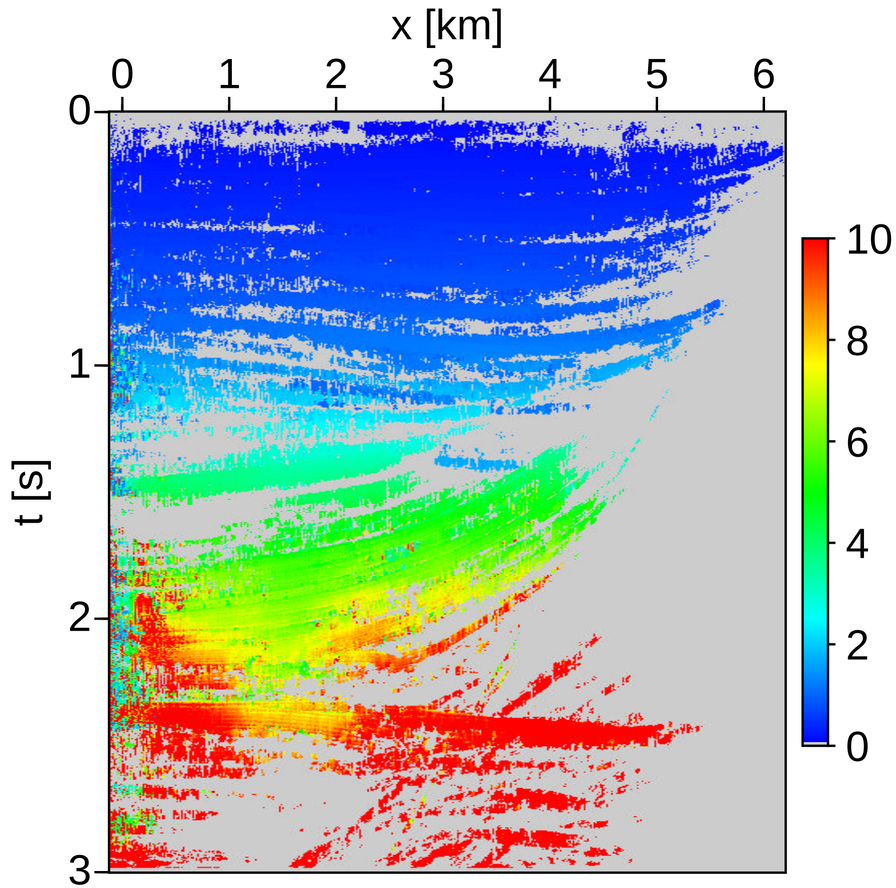


CRS stack section

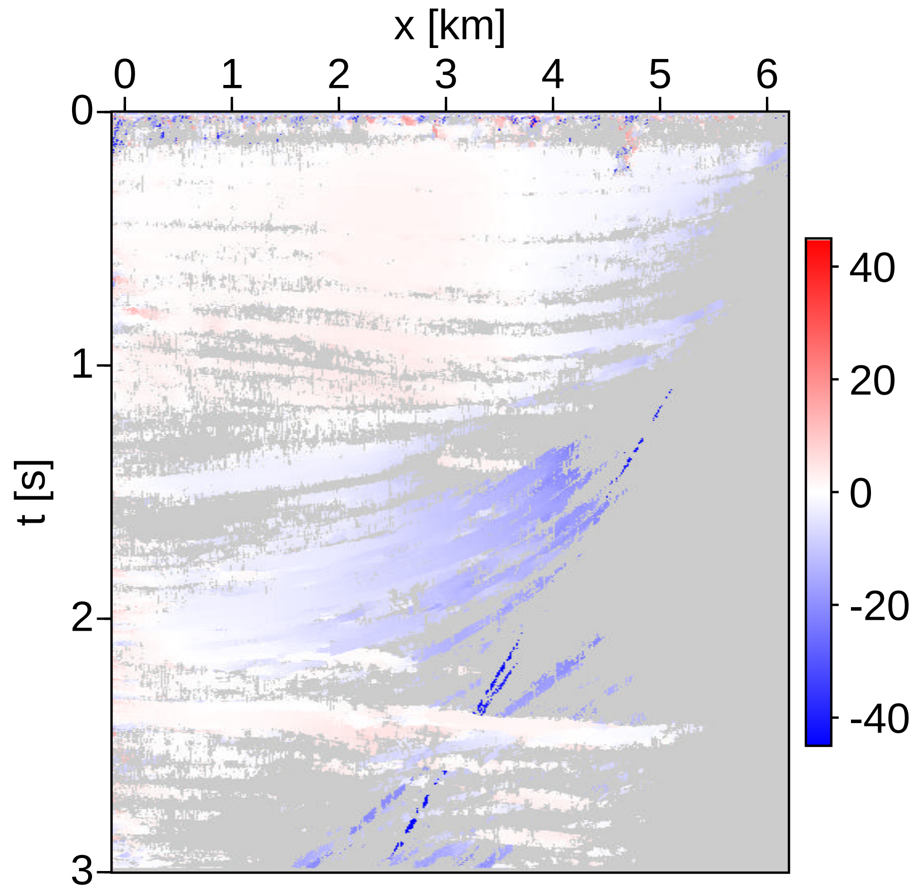


coherence section (semblance)

Real data example



R_{NIP} section [km]



angle section [°]

Real data example

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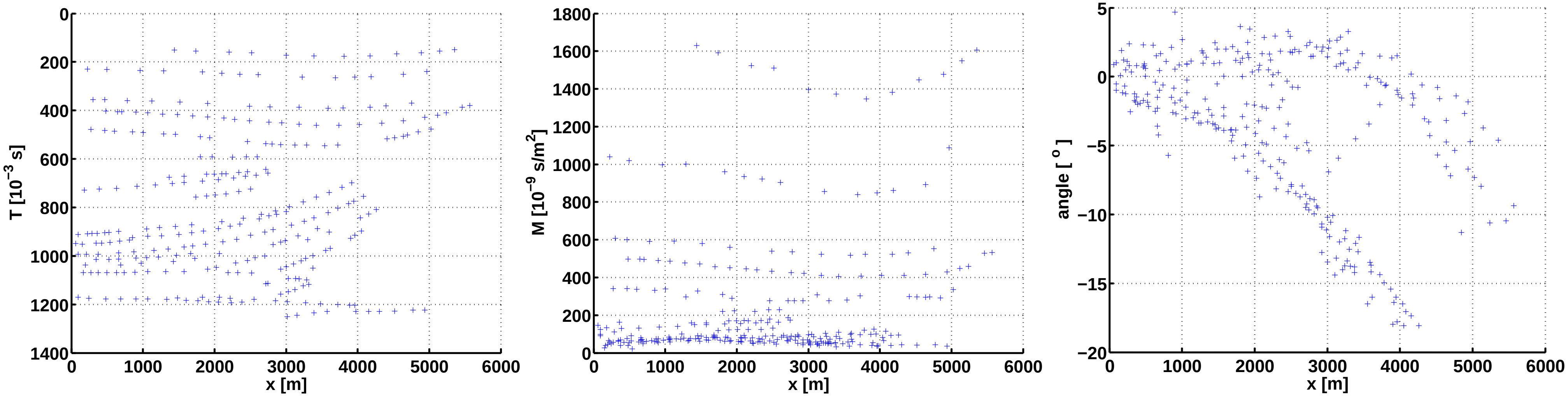

T M α

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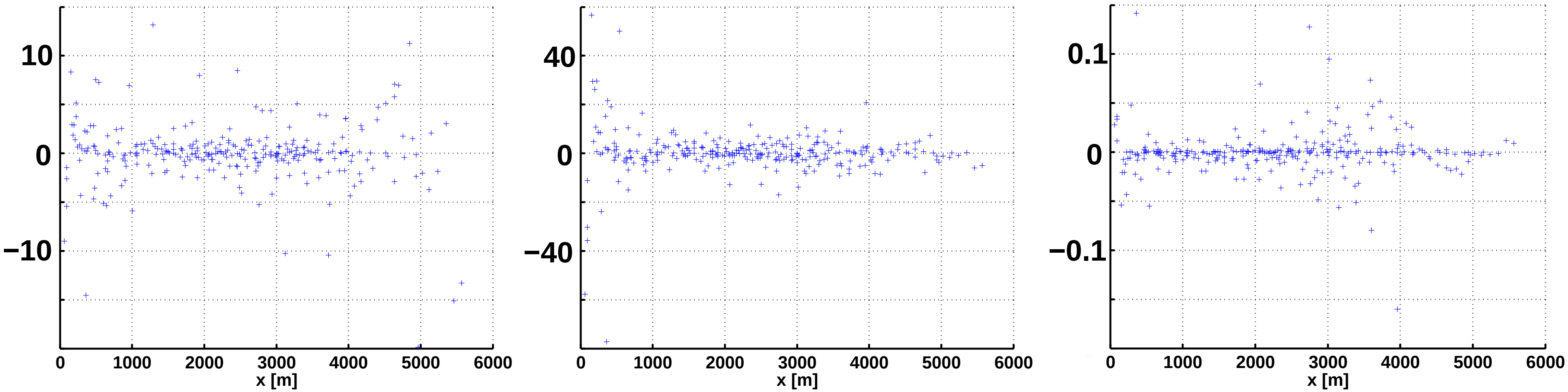

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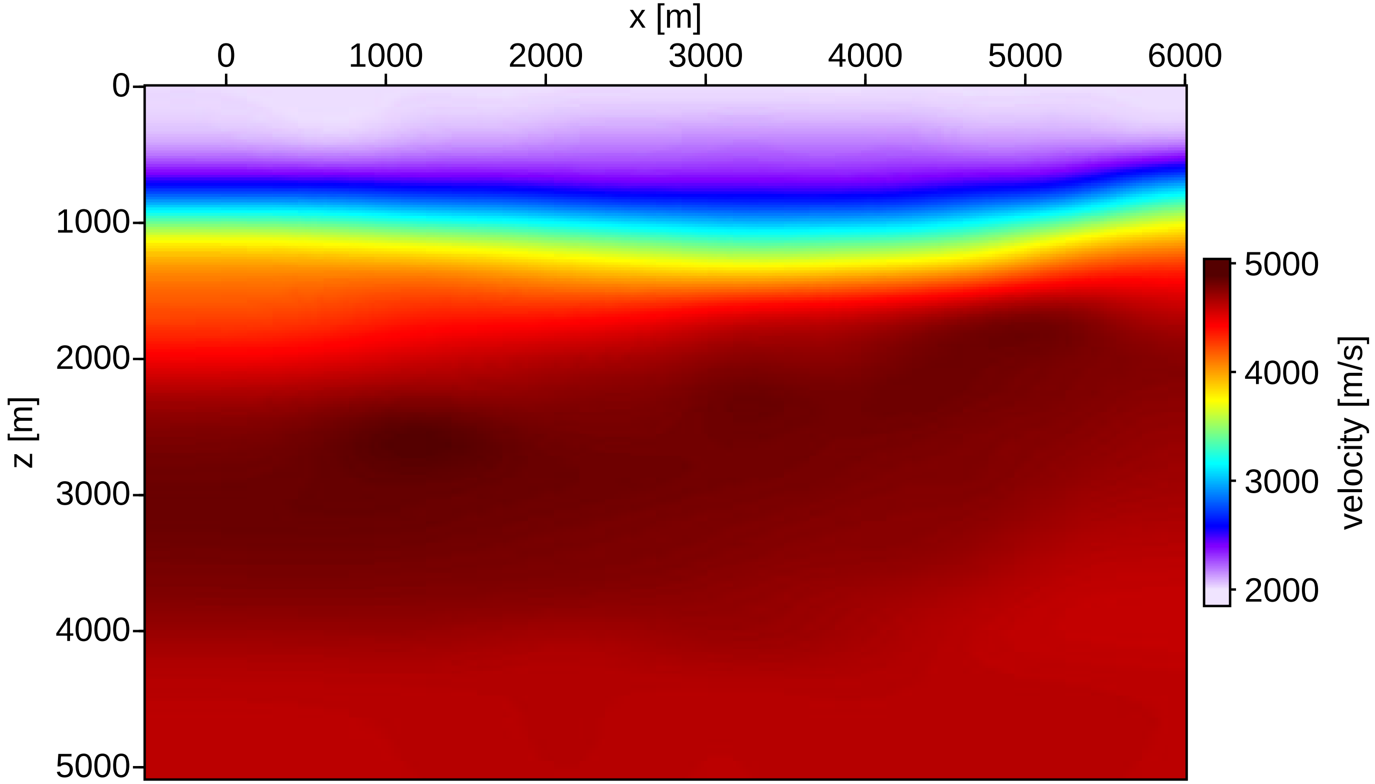

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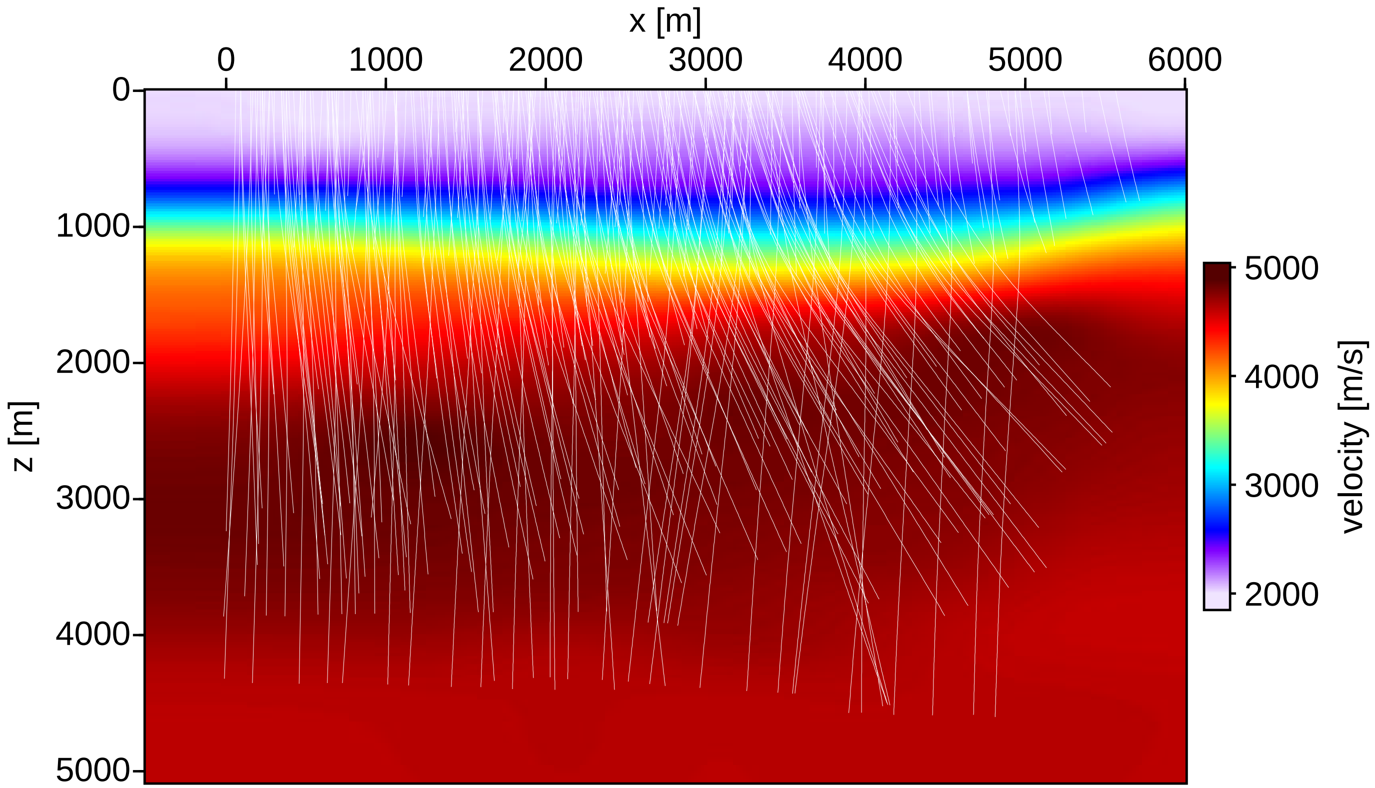
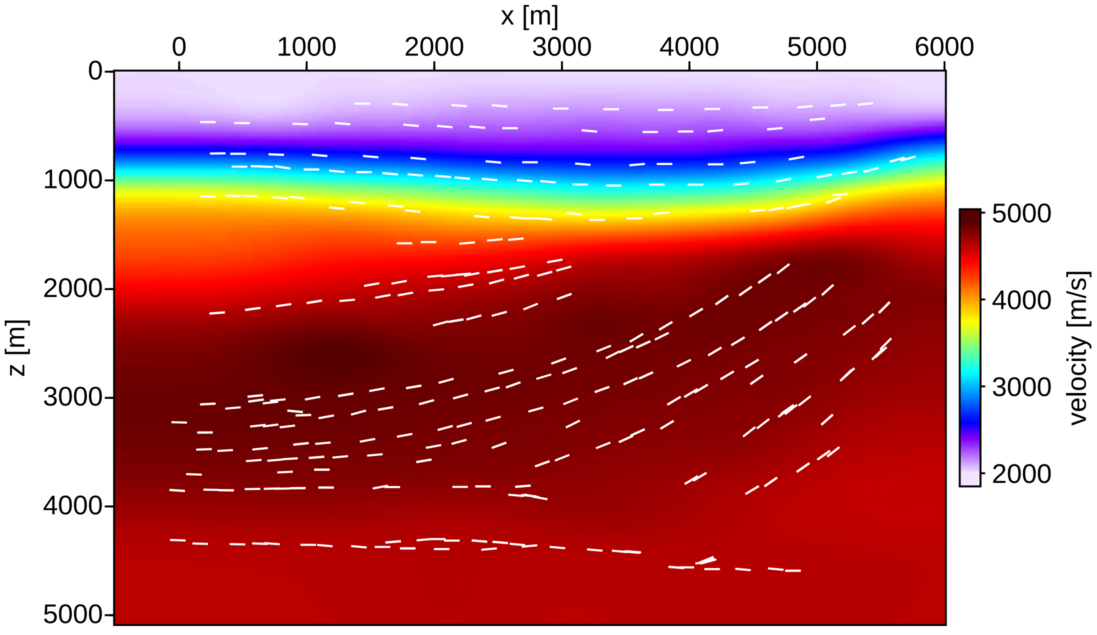
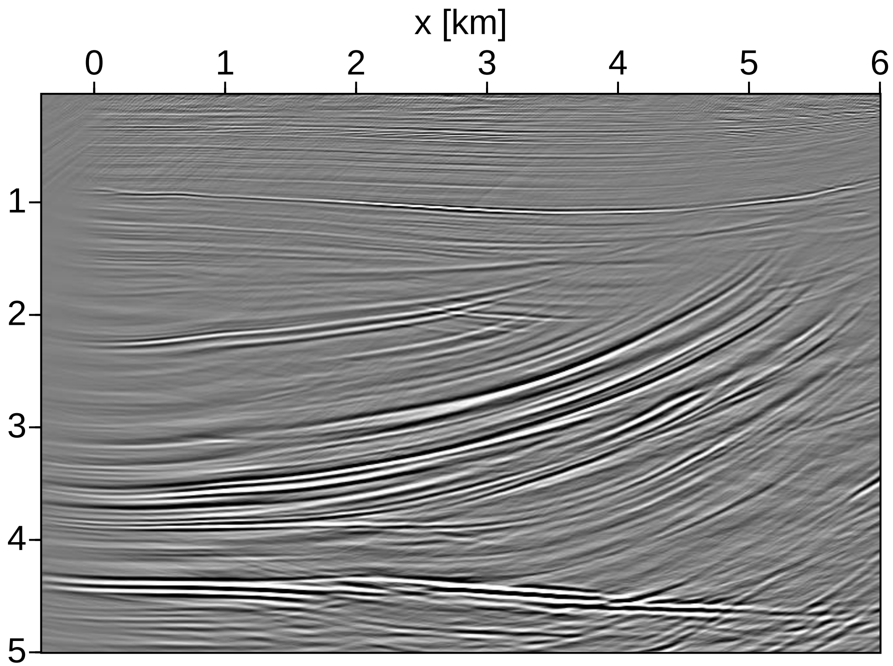


Poststack migration

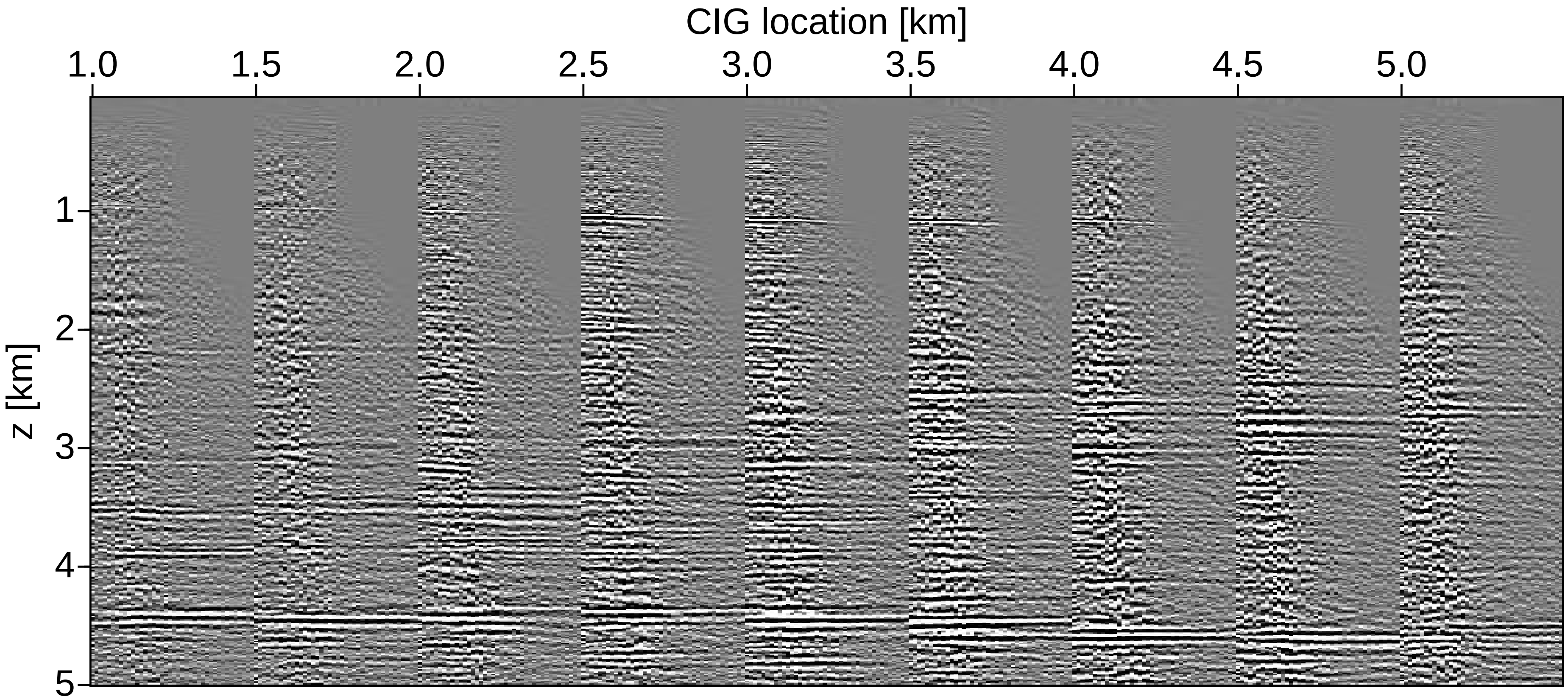


Reconstructed velocity model and dip bars



Poststack migration of CRS stack result

Prestack migration results



common-image gathers (maximum offset=2000m)

3D CRS attributes

$$t^2(\Delta\boldsymbol{\xi},\mathbf{h}) = \left(t_0 + 2\,\mathbf{p}_\xi \cdot \Delta\boldsymbol{\xi}\right)^2 + 2t_0\left(\Delta\boldsymbol{\xi}^T \mathbf{M}_\xi\, \Delta\boldsymbol{\xi} + \mathbf{h}^T \mathbf{M}_h\, \mathbf{h}\right)$$

$$\mathbf{p}_\xi = \tfrac{1}{2}\partial t/\partial\boldsymbol{\xi} = \frac{1}{v_0}(\sin\alpha\cos\psi, \sin\alpha\sin\psi)^T$$

$$\mathbf{M}_h = \tfrac{1}{2}\partial^2 t/\partial\mathbf{h}^2 = \frac{1}{v_0}\mathbf{D}\mathbf{K}_{\text{NIP}}\mathbf{D}^T$$

$$\mathbf{M}_\xi = \tfrac{1}{2}\partial^2 t/\partial\boldsymbol{\xi}^2 = \frac{1}{v_0}\mathbf{D}\mathbf{K}_{\text{N}}\mathbf{D}^T$$

Independent of near-surface velocity v_0 !

3D CRS attributes

$$t^2(\Delta\boldsymbol{\xi},\mathbf{h}) = \left(t_0 + 2\,\mathbf{p}_\xi \cdot \Delta\boldsymbol{\xi}\right)^2$$

$$+2t_0\left(\Delta\boldsymbol{\xi}^T \mathbf{M}_\xi\, \Delta\boldsymbol{\xi} + \mathbf{h}^T \mathbf{M}_h\, \mathbf{h}\right)$$

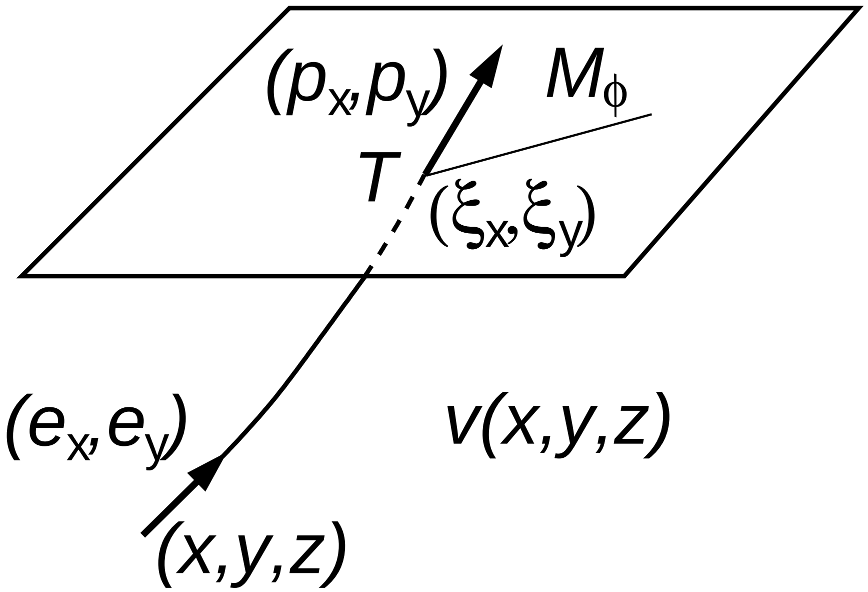
For a tomographic inversion, only

one azimuth ϕ of $\mathbf{M}_h$ is required: M_ϕ!

$\Rightarrow$ Data: $(T, M_\phi, p_{\xi_x}, p_{\xi_y}, \xi_x, \xi_y)$

3D inversion with CRS attributes

Data and model components



- Data:
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$$T = t_0/2$$

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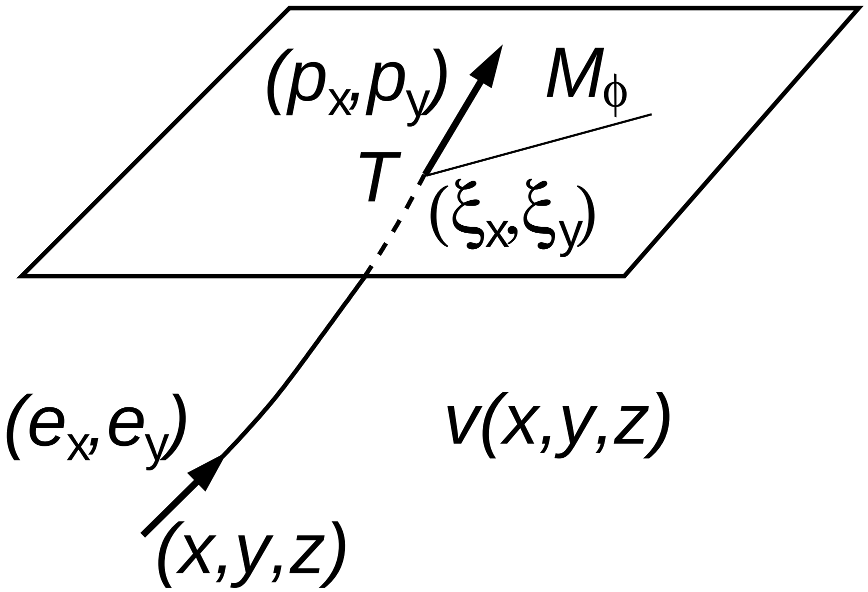

- Data:
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- Model:
  $(x, y, z, e_x, e_y)_i, v_{jkl}$

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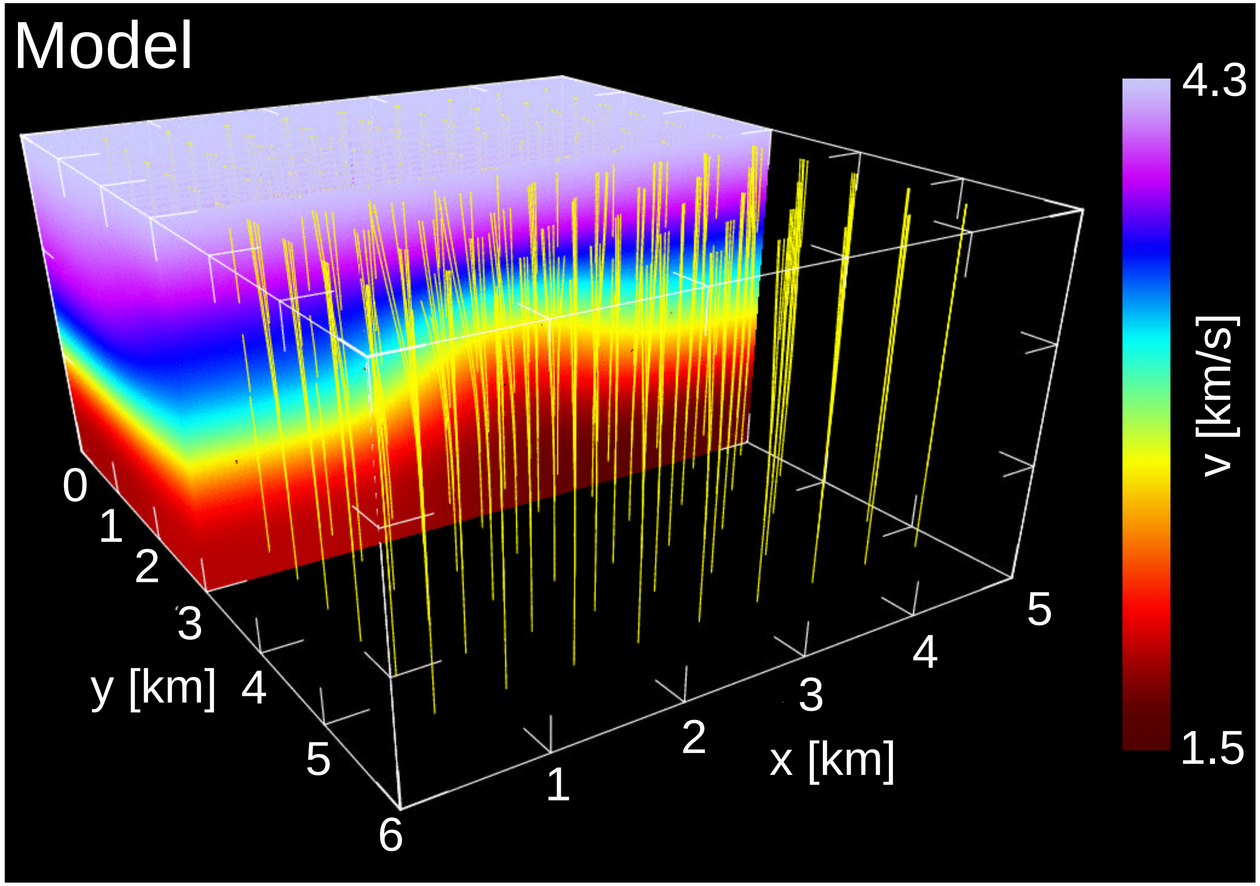
v_{jkl}: B-spline coefficients

3D synthetic example

Cut through original and reconstructed 3D models



original model

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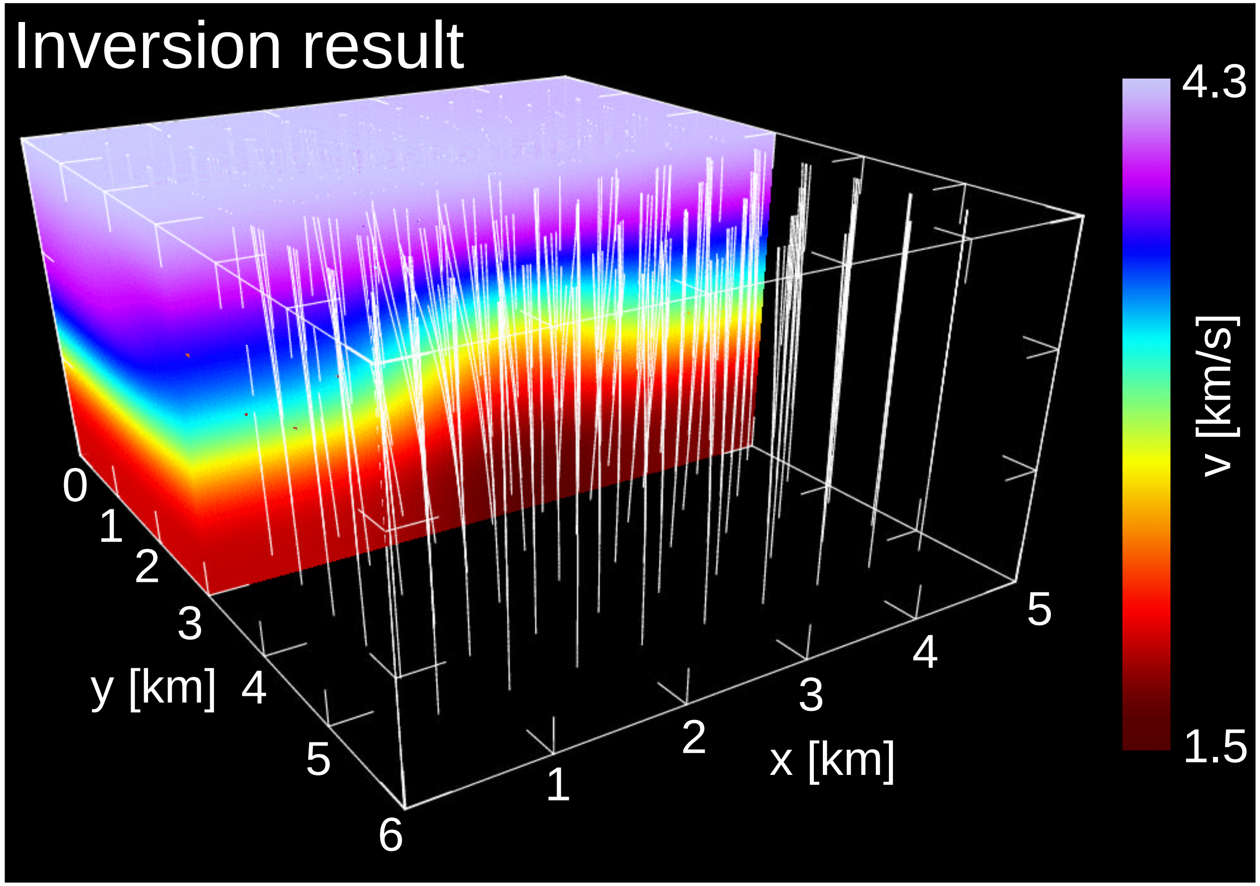

inversion result

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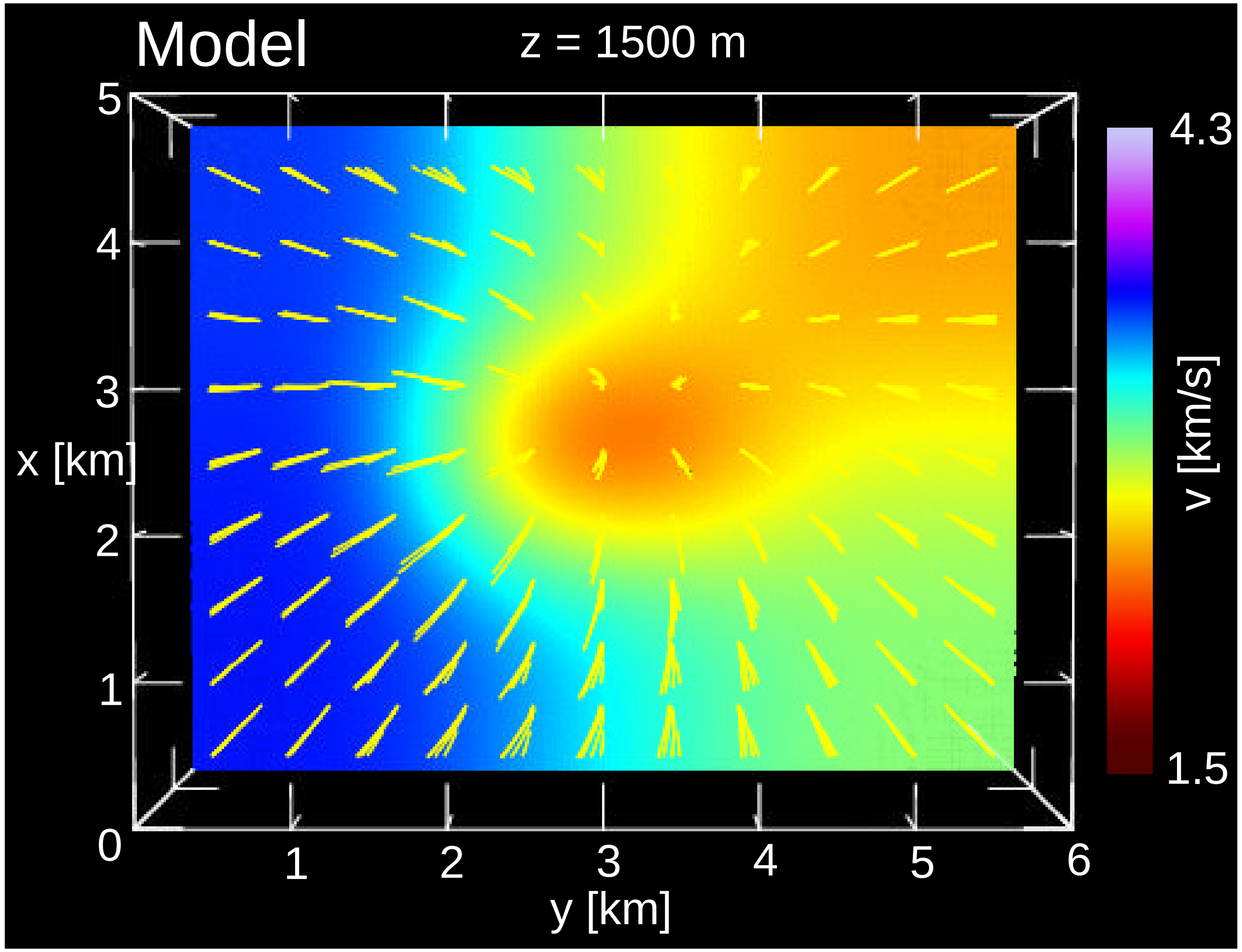
Depth slice at z=1500 m



original model

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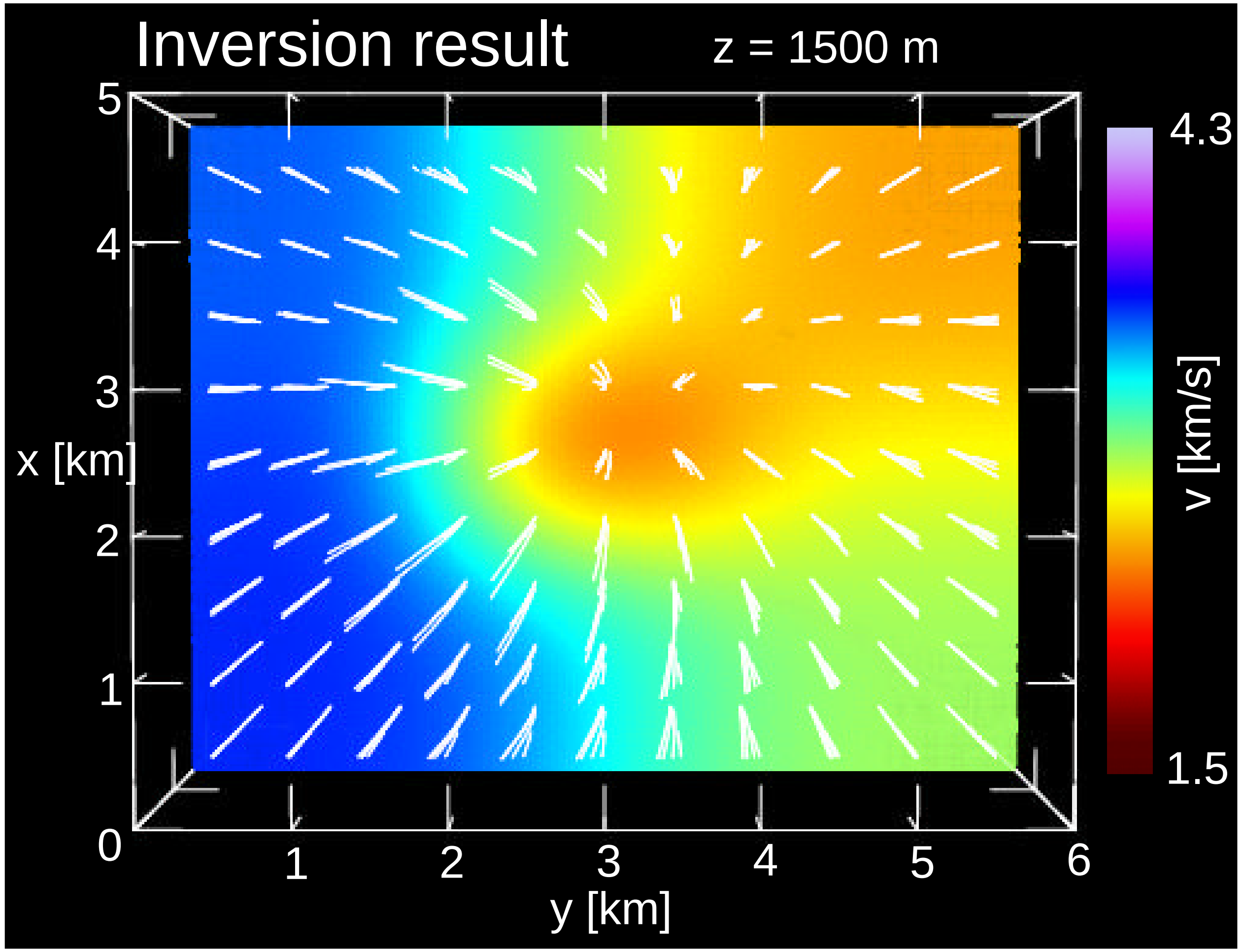

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- Limited lateral variation within CRS aperture (approximately hyperbolic traveltimes)

Conclusions

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- Implementation in 2D and 3D
- Applied to 2D real data

Acknowledgment

This work has been supported by the sponsors of the Wave Inversion Technology (WIT) consortium.